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Merlin and Viviane

By JAMES B. SHAW
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ONCE UPON A TIME many, many years ago there lived the wisest man in the world. Of stately carriage and dignified mien, he was the center of all eyes wherever he appeared. He was so wise that he knew all things and could do the most marvelous works. He lived in a magnificent castle of his own building, for an he wished, a thousand gnomes and elves appeared, and straightway laid stone upon stone faster than eye could follow, so that in only a few moments of time there arose a stately structure covered with the most fascinating towers and spires, parapets and crenellated lace-work. Intricate designs of colored glass filled the windows, and on the walls were traceries of purple and gold, scarlet and emerald. No one could read their meaning, except the wise man himself, for they were really statements about the laws of the universe. When he traveled he need only wish for a carrier, and wonderful birds with long white wings came to take him upon their backs whither he would go; or sometimes the winds would seize him gently and then with a dizzy rush would set him down at his destination. More surprising still, if he desired to live in the world behind the mirror, a simple wish transported him into a new realm that mortal man had never explored before. If he longed for music he waved his hand, and out of the ether came most entrancing harmonies, weird and keen beyond mortal music, harmonies never expressed on the scale of earthly instruments. No octaves were in these compositions, no major thirds, but minor chords which sounded once and were not repeated in the cadenced melody. Indeed he was able to produce music of such complicated form that each note was itself the whole sequence of an endless composition, and as the symphony flowed on, at every moment the ear was charmed by an infinite symphony given instantaneously. At his command the landscape would bloom with flowers of most intricate design. In ordinary flowers the parts are arranged in a winding cycle, but in his flowers there were cycles of cycles. The patterns they made were a delight to the eye. The units that were repeated in many cycles were them-

selves more and more complicated patterns which could not be untangled into simpler ones. Did Merlin feel the need of diversion, a wave of his wand would bring before him a lovely garden filled with beautiful knights and ladies who would produce dance transformations of a most bewildering variety. In many of these they would be arranged in two opposite sets, and these would interchange over and over but always maintaining their part in the set to which they belonged. In other transformations the dancers would gradually get farther and farther away, becoming smaller and smaller, until having reached a faraway boundary, they would return by a series of the reverse kind.

Merlin was more marvelous still, since he could read the messages from the stars and knew where they would be found in all time to come. The distant nebulae, 72,000,000 light-years away were friends of his as they sped along at more than 7,000 miles a second. And he knew the secrets of the little systems of points called atoms, from which quivering waves spread out through the visible universe by trillions and trillions. He could see the fine silvery cosmic dust flashing through space in all directions and could penetrate to the hearts of the tiny particles whence it darted. Merlin could re-create that which had once happened in the world and make it live again with its actors going through their parts, and he could pass through the dim vistas of the future and bring before him in his Joyous Garden that which is to come. He knew himself as part of that vast organism called the living universe, and he knew himself as also a vast living universe, and could trace the life-histories of the infinitely minute creatures which in him lived and moved and had their being. He could even pass below the realm of that which can be measured, into the realm of the infinitesimal, where an endless series of realms lie within realms. And he knew the magic spell which would keep a process of change going on forever. No material thing held its secret from his sight, and he could create an unlimited wealth of new forms that material things might take.

Merlin was consulted by kings and emperors. When they wished to build bridges, canals, tunnels, or cathedrals, he was the chief engineer. He designed their factories, the machinery which ran inside, then the means of transportation. Into his own realms few could pass, so that for them more ordinary means had to be devised. He told them how to put up their telephone lines, and their wireless broadcasting

antennae. He built airplanes and submarines. He located earthquake foci and predicted the storms that would appear. He managed finances, analysed statistics, and solved problems of population. In fact he was indispensable in all the affairs of the nations.

There lived in the same land as Merlin a fairy named Viviane. She was the daughter of a great scholar and a fairy that lived in one of the lakes. From her father she inherited a very practical nature, and was much given to having things sorted and put away in various places where they could always be found. She used to sit by the lake and dream of the days when she would have an establishment of her own, and her mother used to come up out of the depths of the lake and talk to her and tell her the way to accomplish her purposes. Her mother told her about Merlin and all his wonderful knowledge and skill. And she also told Viviane that when she met Merlin she should undertake to cause him to fall in love with her. Then when the time was ripe she should coax him to reveal to her the secret magic by which a man could be bound fast forever to stay with a woman. Viviane was very ambitious and the program pleased her vanity also. She was attractive and many suitors had tried their luck with her but all had failed to interest her permanently.

In the course of time Merlin rode through the woods of Broceliande, and stopped at a spring to refresh himself with the cool water. After drinking he saw Viviane sitting on the other side of the spring looking at him, and he spoke to her. She asked him who he might be, well knowing however that he was Merlin, though he was disguised as a traveling scholar. He replied that he was a student. She inquired what he studied, and he replied that his was the art of magic. She begged him to show her some of his wonderful powers. He graciously consented and traced his mystical characters upon the grass. The forest glade was at once filled with the most wonderful things Viviane had ever beheld, and she was charmed with their beauty. She begged for the secrets of his magic and he promised that sometime he would reveal them to her. Then he waved his hand and everything vanished but the garden. He went on his way, but a year later passed once again that way to keep his tryst with Viviane.

He found her waiting for him, with an exquisite banquet for two prepared. But the viands, dainty as they were, were somewhat wasted, for he was really becoming much in love with her. She however was, as always, calm and placid as the lake in which her mother

lived. This time she learned some of the secrets of magic, but it was not until the next time Merlin came, when the wild roses were blooming, that she succeeded in finding out the magic charm which would bind him in her power. Under her persuasion he taught her the enchantment that would bind him to her forever. And she used the charm, building for him a tower of air, weaving in the blue of heaven, and wrapping him in fabrics ærial, ethereal, clear, crystalline, and rare.

“Ærial, ethereal,
And crystal clear as glass,
Or mirrored surface of the pool
Before the storm clouds pass,
The elfin charm drave up the air
And scarcely stirred the grass.”

“Ærial, ethereal,
Athwart the gleam and glow,
The airy charm wheeled slowly up,
And slowly, and more slow,
‘Lo, I am Merlin,’ spake a voice,
‘And thus our glories go!’ ”*

It was Merlin who created Arithmetic. He started by creating the system of natural numbers, perhaps to account for the various rhythms that impressed him in his daily life. Then he created the rational numbers. These were not included in the system of natural numbers, were not to be found in nature. They were the product of a magician's wand. Then Merlin caused the irrational numbers to come into being. These were not derived from the rational numbers, for they depend upon a notion which the rational numbers do not need, namely that of *limit*. Beyond these Merlin has created the non-Archimedean numbers, which, like irrationals, are not found in the world of nature. That Merlin will not create some day other numbers, who would believe? That is, provided he be not really spell-bound by Viviane.

Merlin waved his wand and created algebra. This happened when he made new numbers which would satisfy the equations that he studied, most of which had no solutions in the realm of arithmetic. Thence came like a new butterfly the negatives, and after them the four-winged complexes. Many years later in trying to find the sets

* E. Mackinstry

of numbers which would be given as the solutions of equations that contained parameters, he created the entire list of hypernumbers, beginning with quaternions, and continuing to the Dirac numbers of modern physics, and non-associative systems which are yet to be applied to natural problems. Out of arithmetic flows the theory of ensembles, and out of algebra flows the theory of operations. Out of arithmetic comes order and tactic. Out of algebra comes the theory of groups. Arithmetic gives us a universe of distinct objects, while algebra manifolds this world into multitudes of related worlds as if the world of arithmetic had been reflected in magic mirrors over and over. The mirrors themselves have been made more and more numerous till now they are unlimited in number, and the plurality of the worlds has become an infinity.

Merlin looked at the crude drawings of early man and with his power of magic made them exact and refined, then he filled them full of new lines and more and more intricate designs until geometry appeared on the scene. With his discerning eye Merlin saw the lines traced on a cone which he called conic sections, and he studied them a thousand years before they were found in nature. He made other curves, created like the cissoid and the conchoid to enable him to solve his geometric equations. The conchoid furnished the pattern for trisecting an angle, and the cissoid for making cubes which would multiply other cubes. He sensed a general procedure of this kind but it was many centuries before he had learned the magic to bring it about. He also used his notion of limit on his curves and areas and volumes producing the geometric irrational. He even had geometric non-Archimedean objects. Then the day came when he created new geometries, whole new worlds of lines and curves; and not content with three-way space, he expanded his universe into space of any number of ways in which he could move and draw his designs. Then Merlin learned a new magic and could make the world of space disappear and become the world of algebra or the world of algebra could become the world of space. In fact it was only one world which appeared in two different forms. The world of arithmetic was also brought into this one world of algebra-geometry at the same time, and the enchantment was very marvelous.

Merlin learned the magic of the limiting processes, and Calculus came to build castles beyond the unending succession of finite steps. Continuity of the arithmetic kind, was set alongside the geometric

continuity; the flow of points on curves alongside the flow of points on other curves; functionality appeared as if magic had created a magician who could turn one object into another object, even of a different kind. The walls and materials of the castles disappeared and the structures made of invariants upon which the geometric worlds were hung became visible. By touching the proper spring a group of transformations rolled the walls around like stage scenery. Even the limiting processes led to new equations for which new functions had to be created and the wand of Merlin was kept active. He had learned now that all the various objects he brought forth were manifestations of the same underlying spirit of mathematics in all their apparently diverse forms.

The way that Merlin created is stated definitely in one place. He says, in brief, that he was trying to show that he could not create a set of functions of the kind called nowadays fuchsian functions. But one night after a rather sleepless time he learned the magic for this kind of creation based upon the hypergeometric series as type. Then he tried to create these functions as quotients of functions similar to the theta functions, and by following the analogy of the elliptic functions, he saw how to do the similar thing for the fuchsian functions. A little later while on a geological trip, he saw the fuchsian functions had turned into geometric properties. After some months while resting at the seashore, he saw that the geometric properties had become the properties of indefinite ternary quadratic forms, and then he was able to create, with this new knowledge, a much more extensive class of the fuchsian functions, with the corresponding theta fuchsian functions. Thus the whole new flora of automorphic functions bloomed in the Joyous Garden. Only one kind was lacking, and one day he found this blooming on a boulevard in Paris. This is one instance in thousands of the way that Merlin comes to develop his magic.

Viviane many centuries ago had interested herself in the prophets, but they were quite indifferent to her charms. Her ideas of hard and fast rules did not appeal to them. Then she was interested in the philosophers. They made much of her, but as she could not control the starting points of their journeys she often arrived at a terminus she did not fancy. She also turned her attentions to the theologians, and here she succeeded in achieving a certain amount of real domination. However, the realm slipped away from her control in the course of the ages. Then she turned to the scientists and

seemed to be destined to be Queen of the world, until the day dawned when it became apparent that Science itself derived most of its resources from the magic of Merlin. So Viviane turned to Merlin with the intention of captivating him.

Now Merlin had not been insensible to the charms of Viviane during his long career as a magician. He had wooed her mildly in the days of Euclid, and through all the centuries following he had offered her no small share of his marvelous worlds. Their ordering and arrangement he had designed to please her and even when he had flown on wings she wist not of, he afterwards arranged paths and roads for her delectation. Not so very long ago he created a realm of her own kind far more extensive than she had ever been able to command by her own resources, and gave it to her as an outright gift. It was situated very close to much of his own territory and indeed part of it was his which he generously added to her domain. It was during this visit to the Joyous Garden that she implored him to tell her the secret which would bind him to her forever. This he did in a moment of enthusiasm, and Viviane has been thinking for some years now what a magnificent realm she rules over, and that Merlin the greatest magician the world has ever had lies bound to her irrevocably.

However, though born of a fairy, Viviane does not have much knowledge of Magic, and she has found that though she can wave the magic wand, nothing ever happens. She can indeed explore the hidden parts of her kingdom, she can go into its hidden caves, she can climb its mountains, but it lies there static, frozen, crystallized in solid glass. The old enchantment of the unexpected, the new, the spontaneous, the unsung music that has not existed before, the strange and exotic flowers of worlds that she knows nothing of—all these are beyond her power. She has even found in reading some of Merlin's books that her own powers are largely due to his fancy, and without that to support them they are elusive, and she fears the day may come when her whole kingdom will prove to be merely a dream:—a dream of Merlin's, which, should he fall into deep enough sleep, would vanish into nothingness. Viviane finds that as she explores the kingdom she frequently gets as the result of all her exertions small and trivial rewards, for without Merlin she has no way of knowing where to find the mines of real jewels, she does not know how to select the direct routes to her goals. She never dreams, no vision ever comes

to entrance her view of the future, she never wanders into the wilderness, for she fears she will get lost. Viviane has security and certainty only upon well laid-out roads which must be followed to their inevitable terminations. If the roads fork she can not determine at all which fork to follow, so is obliged often to retrace her steps and take each one in turn. Indeed she is helpless in the selection of the very means by which she will reach the destination. And now that Merlin is imprisoned in a tower of air she wearies of the prisoner she has made.

But the old legend was incomplete, for there is yet one more part to the story. The secret that Merlin whispered to Viviane is successful only so long as Merlin does not desire to create for himself another and new world. The power to create at will which he possesses, no one can bind, not even Merlin, and Viviane is under an illusion. She may dominate any particular world he chooses to be in if he yields to the spell; but when he vanishes into a new world, he eludes her and all the spells she can ever learn. Being a creative agent and free, Merlin might even fool himself for a time into thinking that he had found the final spell for all enchantment, but he will inevitably, as time goes on, spontaneously and unexpectedly produce a new universe. Whatever order exists in any world Merlin has created, may be inspected and its form made clear, and when Viviane has learned the spell of magic which is the essence of this type of order she has power over it.

Indeed there is emerging today under the magic of Merlin, even though he was thought to be bound, a new world of indeterminacy, a world in which there is order in part, but an ever-changing order, the changes being not under order. This is the magic which creates a fluent world, the ever-new flowers always exhibiting varying form, phantom butterflies resting on their petals, frost-crystals whose designs fade into new designs even while we try to catch them, harmonies whose subtle appeal is witchery that is so elusive it does not register at all. The air in Merlin's tower is blowing again, and the wind bloweth where it listeth. The spell is not dissolving, it is the world itself in which the spell is effective which is vanishing. There is no logic to say what it will be a millenium hence and equally there is no logic to say what it was a million years ago.

Merlin's magic is like the improvisation of a great musician, for while the theme runs on, is developed, expressed in every variation, we have no sooner learned it than there creeps in a new note and a new

chord, and then a new theme, which is added to the old. The whole rises like Abt Vogler's in an ethereal cathedral, tower on tower, structure after structure appearing in the music, the work of genii, and becoming grander and more sublime. When one part has been made perfect, a new part already is coming into the symphony, and so on without end. The musician himself does not know whence these arise, even though he follows a system in their development, so he is as much surprised in his divine play as is the hearer. Thus Viviane however successful is obliged to learn the secret spell anew, and in reality she would not have it otherwise. For two phases of mathematics must not be forgotten. One was emphasized not many years ago by Poincaré, in pointing out that if the mathematician stayed inside his ivory tower and merely worked out consequences, he would find himself ultimately engaged in trivialities, going around in circles rather than definitely going somewhere. This phase of mathematics is the one which views the relations of mathematics with the rest of the universe of thought. For instance, consider what the Einstein theory has done for differential geometry. The other phase is that of the new growth of mathematics. It does not consist of artificial flowers of spun glass and wax, kept under a bell-jar. The new postulational systems all originate in some new flowering branch which is alive. One might as well expect to write poetry by assuming a set of words and phrases, and putting them together under a system of rules. Merlin is essential to the Joyous Garden, for without his lively imagination it never would have existed. Mathematics is the daily witness of the creative ability of the human spirit. That it draws necessary conclusions is not a definition, but merely a statement that it is not a whimsical chaos. It has a structural system of invariants, but it is no more a crystallized logical mechanism than is the skylark. And Viviane would not have it otherwise.

So Merlin lies in his tower of air merely waiting for the floating spirit that will beckon him to a new world, a wider universe, the creation of new forms of artistry, a new outblooming of flowers of everlasting beauty.

Aërial, ethereal,
And free as winging bird,
Or tinted orchids high in air
With petals zephyr-stirred,—
"Lo, I am Merlin," speaks a voice,
"And thus is my music heard!"